I think that diffusive slowdown -- which allows steady-state profiles to occur – is controlled by the ratio of the rate of ice/QLL equilibration () to the rate of diffusion ():

(at steady state) (1)

or

(at steady state) (2)

**The 2-variable system**

Let’s consider first the original, 2-variable system. We know that a small diffusion coefficient tends to make small. That leads to steady-state profiles with more steps (“steeper steady-state profiles”), or equivalently, smaller . For example, the simulation shown in the top panel of Fig. 1, employing a small diffusion coefficient (hence small ) is characterized by . By contrast, the simulation shown in the bottom panel, employing a larger diffusion coefficient (hence bigger ), is characterized by .

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| --- | --- |
|  |  |
|  |  |
| **Figure 1**. The 2-variable formulation using LSODA, using (top panel) and (bottom). Taken from Figs. 1 and 2 of *Neshyba Notes, 23 June*. | |

Both parameterizations exhibit an oscillating, “V/” pattern, in which an initial “V” profile evolves into a “” profile, and back.

**Comparison of 1-variable and 2-variable formulations**

In Ella’s 1-variable system, **equilibration is accelerated compared to diffusion**, because that equilibration is built in. According to Eq. 2, this should lead to smaller at steady state. Comparisons are shown in Fig. 2. The 2-variable formulation is characterized by , whereas the 1-variable formulation is characterized by (and shrinking).

|  |  |
| --- | --- |
|  |  |
|  |  |
| **Figure 2**. The 2-variable formulation (left panel) compared to 1-variable formulation (right), both using LSODA integration, and both having . Taken from Fig. 3 of *Neshyba Notes, 23 June*. | |

When the diffusion coefficient is increased even more, Eq. 2 predicts that should increase. Figure 3 shows that when the diffusion coefficient is increased by about an order of magnitude over the results shown in Fig. 2, the wavelength increases to . In this case, the steady state is numerically unstable, however.

|  |  |
| --- | --- |
|  |  |
| **Figure 4**. The 1-variable formulation using DOP853, with an enhanced diffusion coefficient of . Taken from Fig. 4 of *Neshyba Notes, 23 June*. | |

**Moving forward**

The conclusion from the foregoing is that the 1-variable system produces values that are small compared to the 2-variable system, because ice/QLL equilibration is fast compared to diffusion. But what to do about it?

(3)

Here’s the original formulation:

(4)

And here’s a possible modification:

(5)